Machine Design

For

Mechanical Engineering

By

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Syllabus for Machine Design

Design for Static and Dynamic Loading; Failure Theories; Fatigue Strength and the S-N Diagram; Principles of the Design of Machine Elements such as Bolted, Riveted and Welded Joints, Shafts, Spur Gears, Rolling and Sliding Contact Bearings, Brakes and Clutches.

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"Success consists of going from failure to failure without loss of enthusiasm."

(Winston Churchill)

CHAPTER 1 Design for Combined Loading

Learning Objectives

After reading this chapter, you will know:

1. Theories of Failure
2. Maximum Principal or Normal Stress Theory (Rankine's Theory)
3. Maximum Shear Stress Theory (Guest's or Tresca's Theory)
4. Maximum Principal Strain Theory (Saint Venant Theory)
5. Maximum Distortion Energy Theory (Hencky and Von-Mises Theory)

Introduction

A static load is a stationary force or couple applied to a member. To be stationary, the force or couple must be unchanging in magnitude, point or points of application and direction. A static load can produce axial tension or compression, a shear load, a bending load, a torsional load, or any combination of these. To be considered static, the load cannot change in any manner. In most testing of these properties of materials that relate to the stress-strain diagram, the load is applied gradually, to give sufficient time for the strain to fully develop. Furthermore, the specimen is tested to destruction, and so the stresses are applied only once. Testing of this kind is applicable, to what are known as static conditions; such conditions closely approximate the actual conditions to which many structural and machine members are subjected. Another important term in design is "failure". The definition of failure varies depending upon the component and its application. Failure can mean a part has separated into two or more pieces has become permanently distorted, thus ruining its geometry has its reliability downgraded or has its function compromised, whatever the reason.

Theories of Failure

Events such as distortion, permanent set, cracking and rupturing are among the ways that a machine element fails. In uni-axial tension test the failure mechanisms is simple as elongations are largest in the axial direction, so strains can be measured and stresses inferred up to "failure." The "failure" conclusion becomes challenging when the loading is bi-axial or tri-axial. Unfortunately, there is no universal theory of failure for the general case of material properties and stress state. Instead, over the years several hypotheses have been formulated and tested, leading to today's accepted practices. These "practices" as known as theories of failure and are used to analyse the failure of materials.

Structural metal behavior is typically classified as being ductile or brittle, although under special situations, a material normally considered ductile can fail in a brittle manner. Ductile materials are normally classified such that $\varepsilon_f \geq 0.05$ and have an identifiable yield strength that is often the same in compression as in tension ($S_{yt} = S_{yc} = S_y$). Brittle materials, $\varepsilon_f < 0.05$, do not exhibit identifiable yield strength, and are typically classified by ultimate tensile and compressive strengths, S_{ut} and S_{uc}, respectively (where S_{uc} is given as a positive quantity).

The Generally Accepted Theories are:

1. Maximum principal (or normal) stress theory (also known as Rankine's theory)
2. Maximum shear stress theory (also known as Guest's or Tresca's theory)
3. Maximum principal (or normal) strain theory(also known as Saint Venant theory)
4. Maximum strain energy theory(also known as Haigh's theory)
5. Maximum distortion energy theory(also known as Hencky and Von-Mise's theory)

Since ductile material usually fail by yielding i.e., when permanent deformation occurs in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials the limiting strength is the stress at yield point as determined from simple tension test and it is assumed to be equal in tension and compression. For brittle materials the limiting strength is the ultimate stress in tension or compression.

Maximum Principal or Normal Stress Theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (F.O.S) in to consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial system is given by

$\sigma_{t1} = \dfrac{\sigma_{yt}}{F.S}$ For ductile material

$= \dfrac{\sigma_u}{F.S}$ For brittle material

Where, σ_{yt} = Yield points stress in tension as determined from simple tension test, and

σ_u = Ultimate stress

Since the maximum principal stress or stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression this theory is generally used.

Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear at yield reaches a value equal to the shear at yield point in a simple tension test mathematically

$\tau_{max} = \tau_{yt}/F.O.S$ ------------ (i)

Where, $\tau_{max} =$ Maximum shear stress in a bi-axial stress system

$\tau_{yt} =$ Shear stress at yield point as determined from simple tension test

F.O.S = Factor of Safety

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation (i) may be written as;

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.O.S}$$

This theory is mostly used for designing members of ductile material.

Maximum Principal Strain Theory (Saint Venant Theory)

According to this theory the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (strain a yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m.E}$$

∴ According to the above theory

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m.E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.O.S} ------ (i)$$

Where

σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in bi-axial stress system

ε = Strain at yield point as determined from simple tension test

$1/m$ = Poisson's ratio

E = Young's modulus

F. O. S = Factor of safety

From equation no (i), we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.O.S}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (i.e., strain energy at the yield pant) per unit volume as determined from simple tension test.

We know that strain energy per unit volume in a bi-axial stress system

$$U_1 = \frac{1}{2E}\left[(\sigma_{t_1})^2 + (\sigma_{t_2})^2 - \frac{2\sigma_{t_1} \times \sigma_{t_2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.O.S}\right)^2$$

According to the above theory, $U_1 = U_2$

$$\therefore \frac{1}{2E}\left[(\sigma_{t_1})^2 + (\sigma_{t_2})^2 - \frac{2\sigma_{t_1} \times \sigma_{t_2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.O.S}\right)^2$$

$$\text{or } (\sigma_{t_1})^2 + (\sigma_{t_2})^2 - \frac{2\sigma_{t_1} \times \sigma_{t_2}}{m} = \left(\frac{\sigma_{yt}}{F.O.S}\right)^2$$

Maximum Distortion Energy Theory (Hencky and Von-Mises Theory)

According to this theory the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e., distortion energy at yield point) per unit volume as determined from a simple tension test mathematically the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t_1})^2 + (\sigma_{t_2})^2 - \sigma_{t_1} \times \sigma_{t_2} = \left(\frac{\sigma_{yt}}{F.O.S}\right)^2$$

This theory is mostly used fir ductile materials in place of maximum strain energy theory.

Solved Examples

Example 1

Find the maximum principal stress developed in a cylindrical shaft, 8 cm in diameter and subjected to a bending moment of 2.5 kNm and a twisting moment of 4.2 kNm. If the yield stress of the shaft material is 300 MPa. Determine the factor of safety of the shaft according to the maximum shearing stress theory of failure.

Solution:

Given: d = 8 cm = 0.08 m; M = 2.5 kNm = 2500 Nm; T = 4.2 kNm = 4200 Nm

$\sigma_{yield}\ (\sigma_{yt}) = 300$ MPa $= 300$ MN/m^2

Equivalent torque, $T_e = \sqrt{M^2 + T^2}$

$= \sqrt{(2.5)^2 + (4.2)^2} = 4.888$ kNm

Maximum shear stress developed in the shaft,

$$\tau_{max} = \frac{16T_e}{\pi d^3} = \frac{16 \times 4.888 \times 10^3}{\pi \times (0.08)^3} \times 10^{-6}\ \text{MN/m}^2 = 48.62\ \text{MN/m}^2$$

$$\text{Permissible shear stress} = \frac{300}{2} = 150\ \text{MN/m}^2$$

$$\therefore \text{Factor safety} = \frac{150}{48.62} = 3.085$$

Example 2

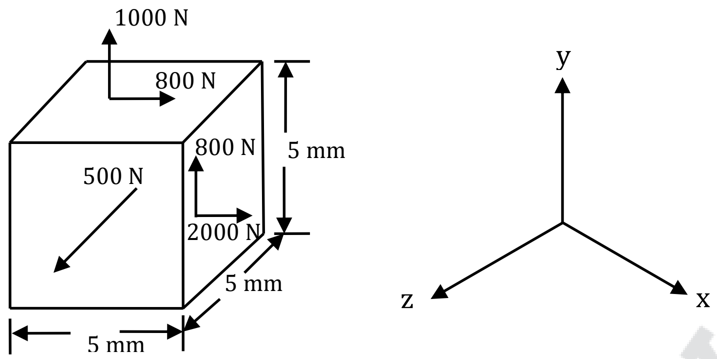
A cube of 5mm side is loaded as shown in Figure.

(i) Determine the principle stresses σ_1, σ_2 and σ_3.

(ii) Will the cube yield if the yield strength of the material is 70 MPa? Use Von-Mises theory.

Solution:

Yield strength of the material, $\sigma_{et} = 70$ MPa $= 70$ MN/m^2 or 70 N/mm^2



(i) Principal stresses σ_1, σ_2 and σ_3:

$$\sigma_x = \frac{2000}{5 \times 5} = 80\ \text{N/mm}^2$$

$$\sigma_y = \frac{1000}{5 \times 5} = 40\ \text{N/mm}^2$$

$$\sigma_z = \frac{500}{5 \times 5} = 20\ \text{N/mm}^2$$

$$\tau_{xy} = \frac{800}{5 \times 5} = 32\ \text{N/mm}^2$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{80+40}{2} \pm \sqrt{\left(\frac{80-40}{2}\right)^2 + (32)^2}$$

$$= 60 \pm \sqrt{(20)^2 + (32)^2} = 97.741,\ 22.26$$

$\therefore\ \sigma_1 = 97.74\ \text{N/mm}^2$ or 97.74 MPa

$\sigma_2 = 22.26\ \text{N/mm}^2$ or 22.26 MPa

$\sigma_3 = \sigma_z = 20\ \text{N/mm}^2$ or 20 MPa

(ii) Will the cube yield?

According to Von-Misses yield criteria, yielding will occur if

$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\sigma_{yt}^2$

Now, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$

$\Rightarrow (97.74 - 22.26)^2 + (22.26 - 20)^2 + (20 - 97.74)^2$

$\Rightarrow 5697.2 + 5.1 + 6043.5 = 11745.8$ ... (i)

$2\sigma_{yt}^2 = 2 \times (70)^2 = 9800$ ... (ii)

Since 11745.8 (i) > 9800, (ii) Hence yielding will occur

Example 3

A thin-walled circular tube of wall thickness t and min radius r is subjected to an axial load P and torque T in a combined tension-torsion experiment.

(i) Determine the state of stress existing in the tube in terms of P and T.

(ii) Using Von-Mises – Henky failure criteria show that failure takes place when $\sqrt{\sigma_x^2 + 3\tau_{xy}^2}$, where σ_0 is the yield stress in uniaxial tension, σ_x and τ_{xy} are respectively the axial and torsional stresses in the tube.

Solution:

Min radius of the tube = r

Wall thickness of the tube = t

Axial load = P

Torque = T

(i) **The State of Stress in the Tube**

Due to axial load, the axial stress in the tube $\sigma_x = \dfrac{P}{2\pi r t}$

Due to torque, shear stress,

$$\tau_{xy} = \frac{Tr}{J_p} = \frac{Tr}{2\pi r^3 t} = \frac{T}{2\pi r^2 t}$$

$J_p = \frac{\pi}{2}\{(r+t)^4 - r^4\} = 2\pi r^3 t$ – neglecting t^2 higher power of t

$\therefore$ The state of stress in the tube is, $\sigma_x = \frac{P}{2\pi r t}, \sigma_y = 0, \tau_{xy} = \frac{T}{2\pi r^2 t}$

(ii) **Von Mises-Henky Failure in Tension for 2-Dimensional Stress is**

$$\sigma_0^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

In this case, $\sigma_1 = \dfrac{\sigma_x}{2} + \sqrt{\dfrac{\sigma_x^2}{4} + \tau_{xy}^2}$ and

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} \quad (\because \sigma_y = 0)$$

$$\therefore \sigma_0^2 = \left[\frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}\right]^2 + \left[\frac{\sigma_x}{2} - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}\right]^2 - \left[\frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}\right]\left[\frac{\sigma_x}{2} - \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}}\right]$$

$$= \left[\frac{\sigma_x^2}{4} + \frac{\sigma_x^2}{4} + \tau_{xy}^2 + 2\,\frac{\sigma_x}{2}\sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}\right]$$

$$+ \left[\frac{\sigma_x^2}{4} + \frac{\sigma_x^2}{4} + \tau_{xy}^2 - 2\,\frac{\sigma_x}{2}\sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}\right] - \left[\frac{\sigma_x^2}{4} - \frac{\sigma_x^2}{4} - \tau_{xy}^2\right]$$

$$= \sigma_x^2 + 3\tau_{xy}^2$$

$$\sigma_0 = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \text{ Proved}$$

Example 4

Compute factor of safety, based upon the distortion-energy theory, for stress elements at A and B of the member shown in the figure. This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces F = 0.55 kN, P = 8.0 kN and
T = 30 N. m., S_y = 280 MPa